

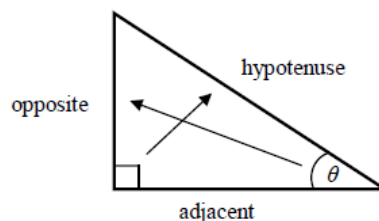
# Trigonometry in right-angled triangles

## A LEVEL LINKS

Scheme of work: 4a. Trigonometric ratios and graphs

### Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle  $\theta$  is called the opposite
  - the side next to the angle  $\theta$  is called the adjacent.

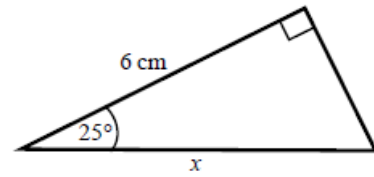


- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle  $\theta$ ,  $\sin \theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle  $\theta$ ,  $\cos \theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle  $\theta$ ,  $\tan \theta = \frac{\text{opp}}{\text{adj}}$
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions:  $\sin^{-1}$ ,  $\cos^{-1}$ ,  $\tan^{-1}$ .
- The sine, cosine and tangent of some angles may be written exactly.

	0	30°	45°	60°	90°
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

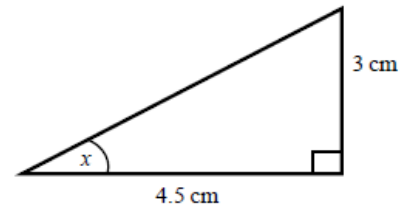
## Examples

**Example 1** Calculate the length of side  $x$ .  
Give your answer correct to 3 significant figures.



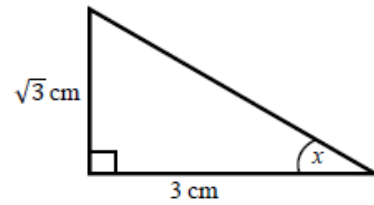
<p> <math display="block">\cos \theta = \frac{\text{adj}}{\text{hyp}}</math> <math display="block">\cos 25^\circ = \frac{6}{x}</math> <math display="block">x = \frac{6}{\cos 25^\circ}</math> <math display="block">x = 6.620\ 267\ 5\dots</math> <math display="block">x = 6.62\ \text{cm}</math> </p>	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the adjacent and the hypotenuse so use the cosine ratio.</li> <li>3 Substitute the sides and angle into the cosine ratio.</li> <li>4 Rearrange to make <math>x</math> the subject.</li> <li>5 Use your calculator to work out <math>6 \div \cos 25^\circ</math>.</li> <li>6 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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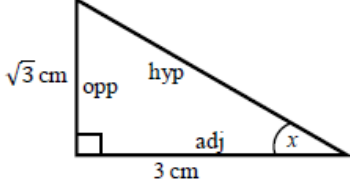
**Example 2** Calculate the size of angle  $x$ .  
Give your answer correct to 3 significant figures.



<p> <math display="block">\tan \theta = \frac{\text{opp}}{\text{adj}}</math> <math display="block">\tan x = \frac{3}{4.5}</math> <math display="block">x = \tan^{-1} \left( \frac{3}{4.5} \right)</math> <math display="block">x = 33.690\ 067\ 5\dots</math> <math display="block">x = 33.7^\circ</math> </p>	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the opposite and the adjacent so use the tangent ratio.</li> <li>3 Substitute the sides and angle into the tangent ratio.</li> <li>4 Use <math>\tan^{-1}</math> to find the angle.</li> <li>5 Use your calculator to work out <math>\tan^{-1}(3 \div 4.5)</math>.</li> <li>6 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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**Example 3** Calculate the exact size of angle  $x$ .

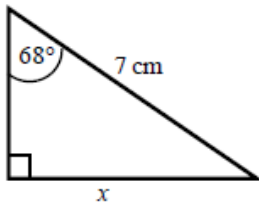


 <p> <math>\tan \theta = \frac{\text{opp}}{\text{adj}}</math>  <math>\tan x = \frac{\sqrt{3}}{3}</math>  <math>x = 30^\circ</math> </p>	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides.</li> <li>2 You are given the opposite and the adjacent so use the tangent ratio.</li> <li>3 Substitute the sides and angle into the tangent ratio.</li> <li>4 Use the table from the key points to find the angle.</li> </ol>
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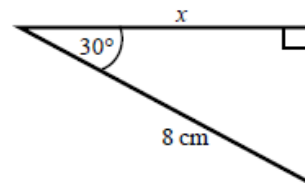
## Practice

1 Calculate the length of the unknown side in each triangle. Give your answers correct to 3 significant figures.

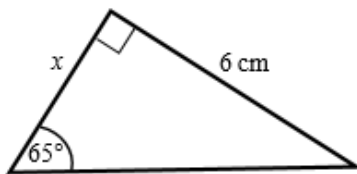
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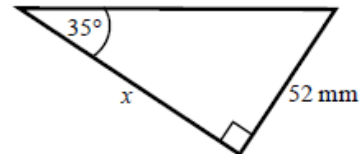
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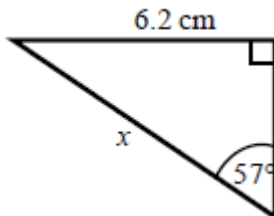
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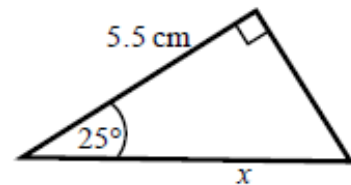
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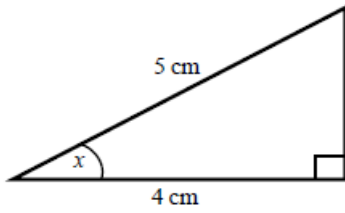


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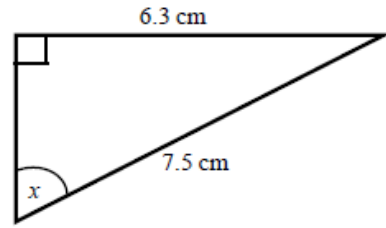


- 2 Calculate the size of angle  $x$  in each triangle. Give your answers correct to 1 decimal place.

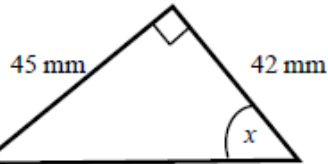
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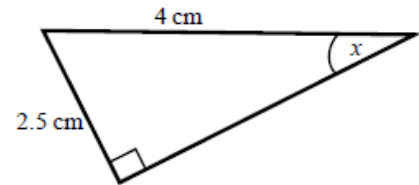
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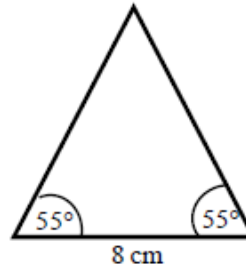
d



- 3 Work out the height of the isosceles triangle. Give your answer correct to 3 significant figures.

**Hint:**

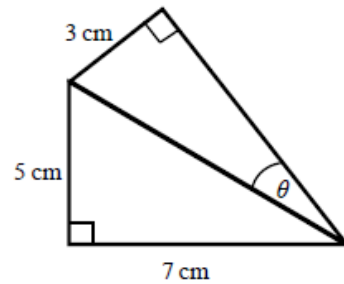
Split the triangle into two right-angled triangles.



- 4 Calculate the size of angle  $\theta$ . Give your answer correct to 1 decimal place.

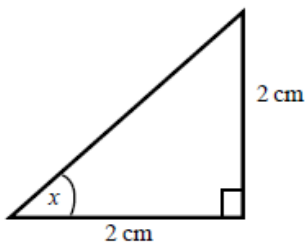
**Hint:**

First work out the length of the common side to both triangles, leaving your answer in surd form.

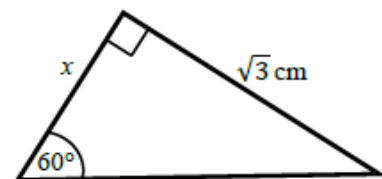


- 5 Find the exact value of  $x$  in each triangle.

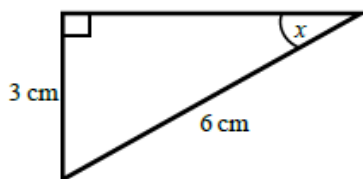
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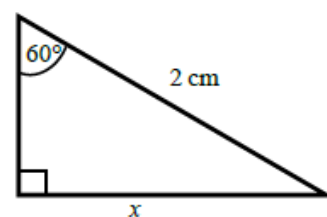
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# The cosine rule

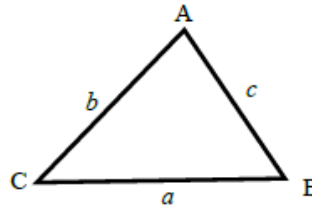
## A LEVEL LINKS

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:** Pure Year 1, 9.1 The cosine rule

## Key points

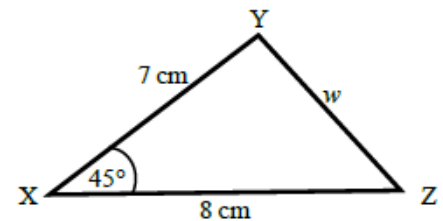
- $a$  is the side opposite angle  $A$ .
- $b$  is the side opposite angle  $B$ .
- $c$  is the side opposite angle  $C$ .



- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula  $a^2 = b^2 + c^2 - 2bc \cos A$ .
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ .

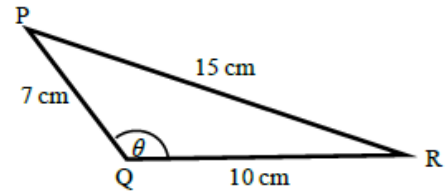
## Examples

**Example 4** Work out the length of side  $w$ .  
Give your answer correct to 3 significant figures.



<p> <math>a^2 = b^2 + c^2 - 2bc \cos A</math>  <math>w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45^\circ</math>  <math>w^2 = 33.804\ 040\ 51\dots</math>  <math>w = \sqrt{33.804\ 040\ 51}</math>  <math>w = 5.81\text{ cm}</math> </p>	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the cosine rule to find the side.</li> <li>3 Substitute the values <math>a</math>, <math>b</math> and <math>A</math> into the formula.</li> <li>4 Use a calculator to find <math>w^2</math> and then <math>w</math>.</li> <li>5 Round your final answer to 3 significant figures and write the units in your answer.</li> </ol>
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**Example 5** Work out the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.

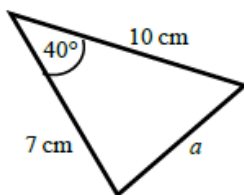


$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos \theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$ $\cos \theta = \frac{-76}{140}$ $\theta = 122.878\ 349\dots$ $\theta = 122.9^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the cosine rule to find the angle.</li> <li>3 Substitute the values <math>a</math>, <math>b</math> and <math>c</math> into the formula.</li> <li>4 Use <math>\cos^{-1}</math> to find the angle.</li> <li>5 Use your calculator to work out <math>\cos^{-1}(-76 \div 140)</math>.</li> <li>6 Round your answer to 1 decimal place and write the units in your answer.</li> </ol>
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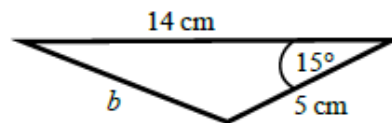
## Practice

6 Work out the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.

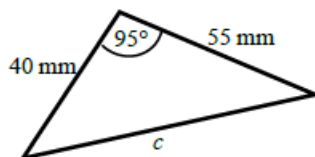
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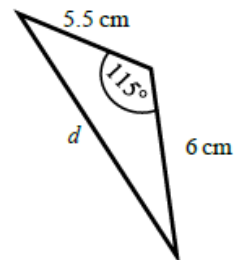
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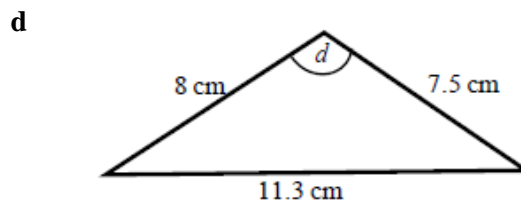
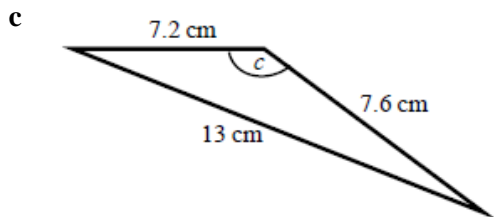
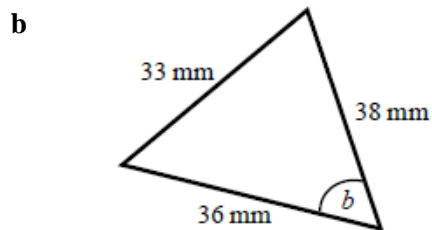
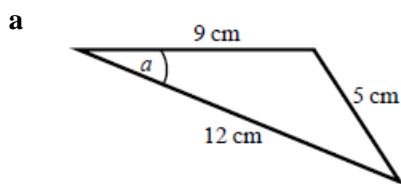
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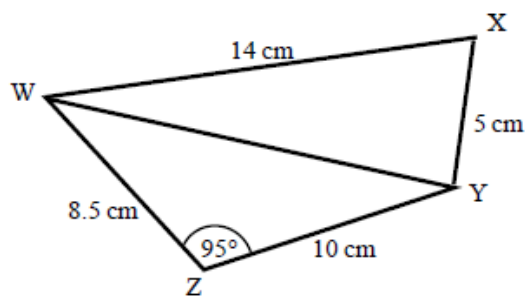
d



- 7 Calculate the angles labelled  $\theta$  in each triangle. Give your answer correct to 1 decimal place.



- 8 a Work out the length of WY. Give your answer correct to 3 significant figures.
- b Work out the size of angle WXY. Give your answer correct to 1 decimal place.



# The sine rule

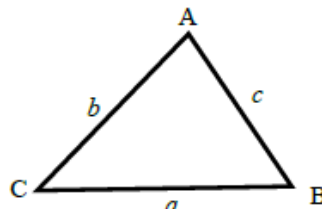
## A LEVEL LINKS

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:** Pure Year 1, 9.2 The sine rule

## Key points

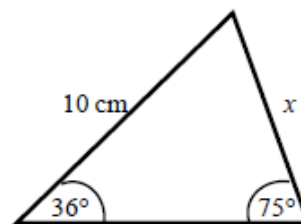
- $a$  is the side opposite angle  $A$ .
- $b$  is the side opposite angle  $B$ .
- $c$  is the side opposite angle  $C$ .



- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula  $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ .
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula  $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ .

## Examples

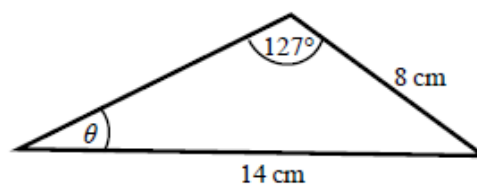
- Example 6** Work out the length of side  $x$ .  
Give your answer correct to 3 significant figures.

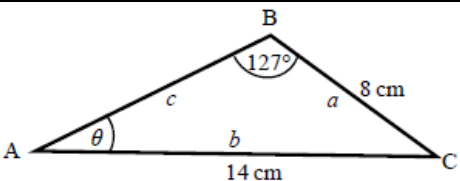


$\frac{a}{\sin A} = \frac{b}{\sin B}$ $\frac{x}{\sin 36^\circ} = \frac{10}{\sin 75^\circ}$ $x = \frac{10 \times \sin 36^\circ}{\sin 75^\circ}$ $x = 6.09 \text{ cm}$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the sine rule to find the side.</li> <li>3 Substitute the values <math>a</math>, <math>b</math>, <math>A</math> and <math>B</math> into the formula.</li> <li>4 Rearrange to make <math>x</math> the subject.</li> <li>5 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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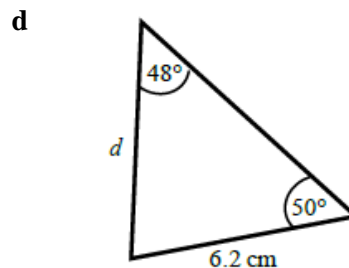
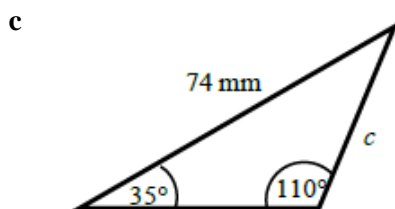
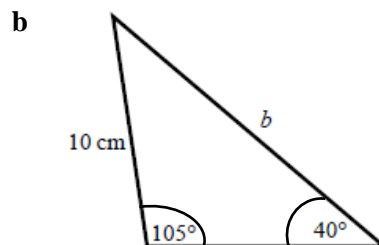
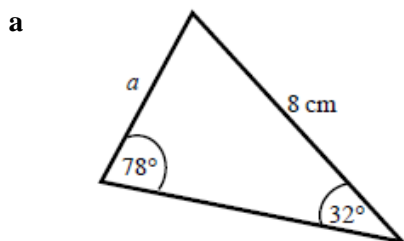
**Example 7** Work out the size of angle  $\theta$ .  
Give your answer correct to 1 decimal place.



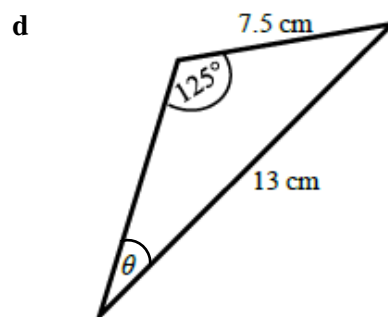
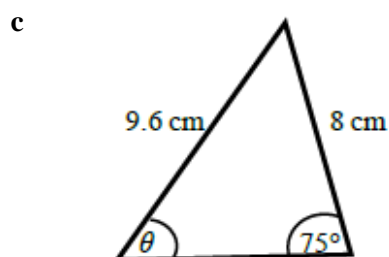
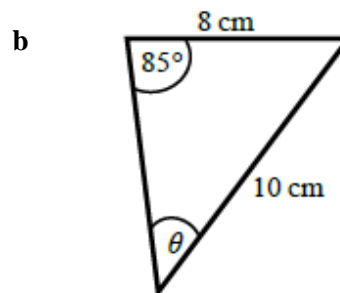
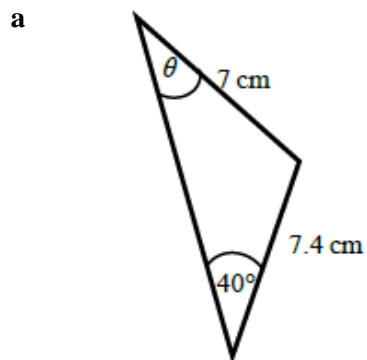
 $\frac{\sin A}{a} = \frac{\sin B}{b}$ $\frac{\sin \theta}{8} = \frac{\sin 127^\circ}{14}$ $\sin \theta = \frac{8 \times \sin 127^\circ}{14}$ $\theta = 27.2^\circ$	<ol style="list-style-type: none"> <li>1 Always start by labelling the angles and sides.</li> <li>2 Write the sine rule to find the angle.</li> <li>3 Substitute the values <math>a</math>, <math>b</math>, <math>A</math> and <math>B</math> into the formula.</li> <li>4 Rearrange to make <math>\sin \theta</math> the subject.</li> <li>5 Use <math>\sin^{-1}</math> to find the angle. Round your answer to 1 decimal place and write the units in your answer.</li> </ol>
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## Practice

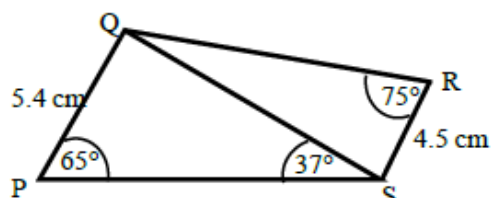
9 Find the length of the unknown side in each triangle.  
Give your answers correct to 3 significant figures.



- 10 Calculate the angles labelled  $\theta$  in each triangle.  
Give your answer correct to 1 decimal place.



- 11 a Work out the length of QS.  
Give your answer correct to 3 significant figures.
- b Work out the size of angle RQS.  
Give your answer correct to 1 decimal place.



# Areas of triangles

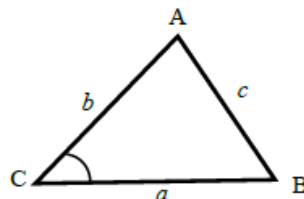
## A LEVEL LINKS

**Scheme of work:** 4a. Trigonometric ratios and graphs

**Textbook:** Pure Year 1, 9.3 Areas of triangles

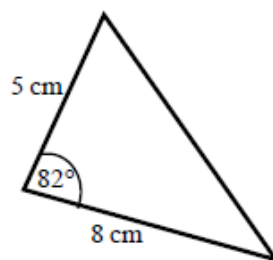
## Key points

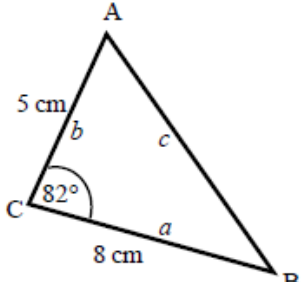
- $a$  is the side opposite angle  $A$ .  
 $b$  is the side opposite angle  $B$ .  
 $c$  is the side opposite angle  $C$ .
- The area of the triangle is  $\frac{1}{2}ab \sin C$ .



## Examples

**Example 8** Find the area of the triangle.

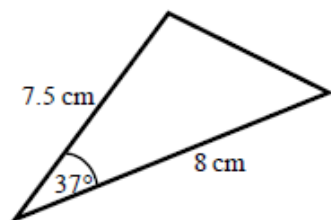


 <p>Area = <math>\frac{1}{2}ab \sin C</math></p> <p>Area = <math>\frac{1}{2} \times 8 \times 5 \times \sin 82^\circ</math></p> <p>Area = 19.805 361...</p> <p>Area = 19.8 cm<sup>2</sup></p>	<ol style="list-style-type: none"> <li>1 Always start by labelling the sides and angles of the triangle.</li> <li>2 State the formula for the area of a triangle.</li> <li>3 Substitute the values of <math>a</math>, <math>b</math> and <math>C</math> into the formula for the area of a triangle.</li> <li>4 Use a calculator to find the area.</li> <li>5 Round your answer to 3 significant figures and write the units in your answer.</li> </ol>
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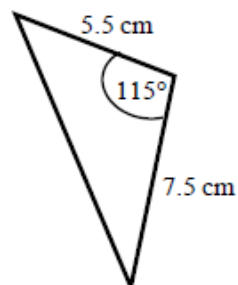
## Practice

- 12 Work out the area of each triangle.  
Give your answers correct to 3 significant figures.

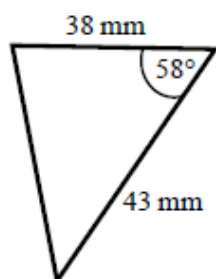
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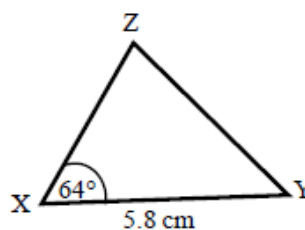
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- 13 The area of triangle XYZ is  $13.3 \text{ cm}^2$ .  
Work out the length of XZ.

**Hint:**

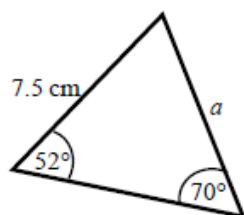
Rearrange the formula to make a side the subject.



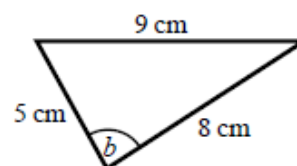
## Extend

- 14 Find the size of each lettered angle or side.  
Give your answers correct to 3 significant figures.

a



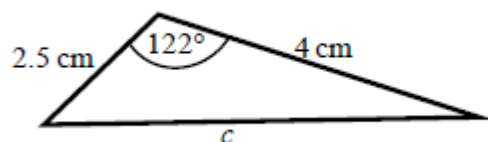
b



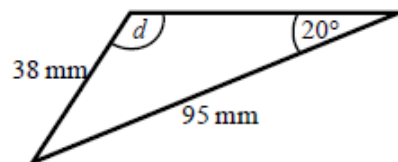
**Hint:**

For each one, decide whether to use the cosine or sine rule.

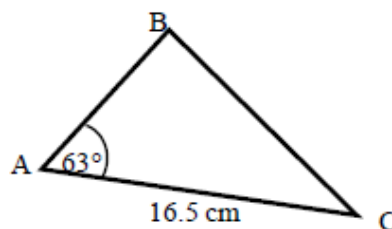
c



d



- 15 The area of triangle ABC is  $86.7 \text{ cm}^2$ .  
 Work out the length of BC.  
 Give your answer correct to 3 significant figures.



## Answers

- |           |                               |                               |                              |                        |
|-----------|-------------------------------|-------------------------------|------------------------------|------------------------|
| <b>1</b>  | <b>a</b> 6.49 cm              | <b>b</b> 6.93 cm              | <b>c</b> 2.80 cm             |                        |
|           | <b>d</b> 74.3 mm              | <b>e</b> 7.39 cm              | <b>f</b> 6.07 cm             |                        |
| <b>2</b>  | <b>a</b> 36.9°                | <b>b</b> 57.1°                | <b>c</b> 47.0°               | <b>d</b> 38.7°         |
| <b>3</b>  | 5.71 cm                       |                               |                              |                        |
| <b>4</b>  | 20.4°                         |                               |                              |                        |
| <b>5</b>  | <b>a</b> 45°                  | <b>b</b> 1 cm                 | <b>c</b> 30°                 | <b>d</b> $\sqrt{3}$ cm |
| <b>6</b>  | <b>a</b> 6.46 cm              | <b>b</b> 9.26 cm              | <b>c</b> 70.8 mm             | <b>d</b> 9.70 cm       |
| <b>7</b>  | <b>a</b> 22.2°                | <b>b</b> 52.9°                | <b>c</b> 122.9°              | <b>d</b> 93.6°         |
| <b>8</b>  | <b>a</b> 13.7 cm              | <b>b</b> 76.0°                |                              |                        |
| <b>9</b>  | <b>a</b> 4.33 cm              | <b>b</b> 15.0 cm              | <b>c</b> 45.2 mm             | <b>d</b> 6.39 cm       |
| <b>10</b> | <b>a</b> 42.8°                | <b>b</b> 52.8°                | <b>c</b> 53.6°               | <b>d</b> 28.2°         |
| <b>11</b> | <b>a</b> 8.13 cm              | <b>b</b> 32.3°                |                              |                        |
| <b>12</b> | <b>a</b> 18.1 cm <sup>2</sup> | <b>b</b> 18.7 cm <sup>2</sup> | <b>c</b> 693 mm <sup>2</sup> |                        |
| <b>13</b> | 5.10 cm                       |                               |                              |                        |
| <b>14</b> | <b>a</b> 6.29 cm              | <b>b</b> 84.3°                | <b>c</b> 5.73 cm             | <b>d</b> 58.8°         |
| <b>15</b> | 15.3 cm                       |                               |                              |                        |